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SIM(2) and Superspace

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ABSTRACT

In this brief note we give a superspace description of the supersymmetric nonlocal Lorentz noninvariant actions recently proposed by Cohen and Freedman. This leads us to discover similar terms for gauge fields.

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1 Introduction

Recently, Cohen and Glashow proposed that certain nonlocal terms that preserve just a SIM(2) subgroup of the Lorentz group may account for neutrino masses without the need to introduce new particles [1]. This proposal was a follow-up to the curious observation that there does not seem to be experimental evidence to rule out dynamics that is governed by this solvable subgroup of the Lorentz group [2].

Subsequently, these neutrino mass terms were supersymmetrized [3]. In this brief note, we give a superspace description of these terms. The resulting superspace action is compact and easy to understand; as we give a superspace measure for these terms, it is easy to write down many generalizations. In particular, we find Majorana like mass terms for chiral superfield fermions as well as mass terms for gauginos; the latter induce nonlocal mass terms for the gauge fields, which become local in a lightcone gauge; though superficially one might think that these terms provide an alternative to the Higgs mechanism, we argue that this is not the case¹.

We note that the SIM(2)-symmetric but Lorentz-noninvariant terms have some resemblance to boundary terms; it would be very interesting if they could be given such an interpretation.²

2 Four component notation

The SIM(2) group is the subgroup of the four dimensional Lorentz group that preserves a fixed null vector up to rescalings; a nice summary of all the relevant facts that we use can be found in [3]. In this section, we present two forms of the SIM-invariant dynamics in superspace. The first is formulated in terms of a constant null vector n ; the second is formulated in terms of a constant spinor ζ .

2.1 Null vector formalism

We begin with a few preliminaries. In addition to the given null vector n , we may choose a second null vector \tilde{n} that obeys $n \cdot \tilde{n} = 1$. Then, since

$$n \cdot \tilde{n} = 1 \Rightarrow \{\not{n}, \not{\tilde{n}}\} = 2 \quad , \quad \left(\frac{\not{n}\not{\tilde{n}}}{2}\right)^2 = \left(\frac{\not{n}\not{\tilde{n}}}{2}\right) \quad , \quad etc., \quad (2.1)$$

we can split any spinor ψ into $\frac{1}{2}\not{n}\not{\tilde{n}}\psi + \frac{1}{2}\not{\tilde{n}}\not{n}\psi$ projections; furthermore, the constraint $\not{n}\epsilon = 0$ has the obvious solution $\epsilon = \not{n}\eta$ for an arbitrary spinor η .

¹Cohen and Glashow have independently considered these terms [4].

²K. Skenderis has pointed out that the intersection of a 3-brane and a 7-brane can be interpreted as giving rise to an effective action with a bulk and a boundary term that both fill the world-volume of the 3-brane.

We keep the Lorentz-invariant part of the action for the chiral superfield Z in full superspace:

$$S_{full} = \int d^4\theta Z \bar{Z} + \left(\int d^2\theta W(Z) + c.c. \right) ; \quad (2.2)$$

To write the Lorentz-breaking term, we split the spinor derivative D into the piece that we keep, $d \equiv \frac{1}{2} \not{n} \not{D}$ and the piece that we expand in: $q \equiv \frac{1}{2} \not{q} \not{D}$. The SIM-superspace algebra of the spinor derivatives is [3]

$$[\bar{\epsilon}_1 d, \bar{d} \epsilon_2] = 2(\bar{\epsilon}_1 \not{n} \epsilon_2)(n \cdot \partial) \quad (2.3)$$

for arbitrary constant spinors ϵ_1, ϵ_2 . We relate the full superspace chiral superfields Z, \bar{Z} obeying

$$D_R Z = 0, \quad D_L \bar{Z} = 0 \quad (2.4)$$

to superfields that we use in SIM-superspace, which are the covariant spinor projections³ of Z, \bar{Z} that are independent of $\frac{1}{2} \not{n} \not{D}$, the conjugate of q : Z, \bar{Z} , and new spinor superfields ψ_L, ψ_R defined by:

$$\psi_L = qZ, \quad \psi_R = q\bar{Z}. \quad (2.5)$$

The SIM-superspace constraints that these superfields obey are

$$d_R Z = 0, \quad d_L \bar{Z} = 0, \quad d_R \psi_L = \{d_R, q_L\} Z, \quad d_L \psi_R = \{d_L, q_R\} \bar{Z}, \quad (2.6)$$

where the anticommutators $\{d_R, q_L\}$ and $\{d_L, q_R\}$ are certain projections of $i\partial$ which we give explicitly in two component form below. The extra SIM(2) invariant but non-Lorentz invariant term is⁴

$$S_{SIM} = m^2 \int (\bar{d} \not{n} d_L) Z \frac{1}{n \cdot \partial} \bar{Z}. \quad (2.7)$$

Neither the SIM-superspace measure nor the SIM-superspace Lagrange density is SIM(2) invariant, but they transform so as to ensure that the action S_{SIM} , which is homogenous in the null vector n , is invariant. Note that the superfield ψ does not enter S_{SIM} . We work out the component form of the action below. We define component fields $Z, \chi_L = d_L Z, \psi_L, F = d_L \psi_L$ and the conjugates $\bar{Z}, \chi_R = d_R \bar{Z}, \psi_R, \bar{F} = d_R \psi_R$, and push in the d 's to find the action. We also need to use the constraints (2.6). A useful identity is:

$$\bar{d} \gamma^\mu d = \tilde{n}^\mu \bar{D} \not{n} D \Rightarrow \bar{d} \not{n} d = \bar{D} \not{n} D. \quad (2.8)$$

³We use the same name for the full superfields and their leading SIM-superspace projections. The SIM-superspace fields are defined by the standard method of covariant projection; equivalently, we may write the explicit θ -expansion: $Z_{full} = Z_{SIM} + \frac{1}{2} \bar{\theta} \not{n} \not{D} \psi_L - \bar{\theta} \not{n} \gamma^\mu \not{D} \theta_L \partial_\mu Z_{SIM}$.

⁴The full superspace measure can be related to the SIM-superspace measure by $\int d^4\theta \propto \int (\bar{d} \not{n} d_L) (\bar{q} \not{n} q_R)$.

2.2 Spinor formalism

The null condition $n \cdot n = 0$ is solved in terms of a single commuting Majorana spinor ζ :

$$n^\mu = \bar{\zeta} \gamma^\mu \zeta . \quad (2.9)$$

Clearly, $\not{n}\zeta = 0$; hence the SIM-supersymmetry transformations are generated by a spinor

$$\epsilon = (a + ib\gamma_5)\zeta , \quad (2.10)$$

where a, b are real *anticommuting* scalar parameters. Similarly, we write \tilde{n} in terms of a second commuting Majorana spinor $\tilde{\zeta}$:

$$\tilde{n}^\mu = \tilde{\bar{\zeta}} \gamma^\mu \tilde{\zeta} , \quad \tilde{\bar{\zeta}} \tilde{\zeta} = 2 . \quad (2.11)$$

Then we write:

$$d_L \equiv \tilde{\zeta}_L (\bar{\zeta} D_L) , \quad d_R \equiv \tilde{\zeta}_R (\bar{\zeta} D_R) , \quad q_L \equiv \zeta_L (\tilde{\bar{\zeta}} D_L) , \quad q_R \equiv \zeta_R (\tilde{\bar{\zeta}} D_R) , \quad (2.12)$$

and the SIM(2) invariant action becomes

$$S_{SIM} = m^2 \int (\bar{\zeta} D_L) (\tilde{\bar{\zeta}} D_R) \left(Z \frac{1}{n \cdot \partial} \bar{Z} \right) . \quad (2.13)$$

3 Two component notation

This can all be made more transparent in two component notation. The only nonvanishing components of n and \tilde{n} can be chosen to be n^{++} and \tilde{n}_{++} , or equivalently, the only nonvanishing components of the commuting spinors ζ and $\tilde{\zeta}$ can be chosen to be $\zeta^+, \bar{\zeta}^+$ and $\tilde{\zeta}_+, \tilde{\bar{\zeta}}_+$. Then the spinor derivatives⁵ d, q are simply:

$$d_+ = D_+ , \quad q_- = D_- , \quad \bar{d}_{\dot{+}} = \bar{D}_{\dot{+}} , \quad \bar{q}_{\dot{-}} = \bar{D}_{\dot{-}} , \quad (3.1)$$

where the undotted indices correspond to left-handed spinors and the dotted indices correspond to right-handed spinors (up to a convention). The algebra of the spinor derivatives is also very simple

$$\{d_+, \bar{d}_{\dot{+}}\} = i\partial_{++} , \quad (3.2)$$

where $\partial_{++} = n \cdot \partial$, $\partial_{--} = \tilde{n} \cdot \partial$, *etc.* The full chiral superfields obey

$$\bar{D}_{\dot{\alpha}} Z = 0 , \quad D_{\alpha} \bar{Z} = 0 , \quad (3.3)$$

⁵Two component spinors $D_\alpha, \bar{D}_{\dot{\alpha}}$ correspond to four component spinors D_L, D_R , respectively.

which leads us to define SIM-superspace superfields

$$Z, \psi_- \equiv q_- Z, \bar{Z}, \bar{\psi}_- \equiv \bar{q}_- \bar{Z}; \quad (3.4)$$

these obey the SIM-superspace constraints

$$\bar{d}_+ Z = d_+ \bar{Z} = 0, \quad \bar{d}_+ \psi_- = i \partial_{-+} Z, \quad d_+ \bar{\psi}_- = i \partial_{+-} \bar{Z}. \quad (3.5)$$

The Lorentz symmetry-breaking term is

$$S_{SIM} = -im^2 \int d_+ \bar{d}_+ \left(Z \frac{1}{\partial_{++}} \bar{Z} \right). \quad (3.6)$$

To go to components, we define $Z, \chi_+ \equiv d_+ Z, \psi_-, F \equiv d_+ \psi_-$ and the complex conjugates (actually, as follows from (3.1,3.4), it makes sense to identify $\chi_+ \equiv \psi_+$, which we do below). Using the constraints on the SIM-superfields Z, ψ and the algebra of spinor derivatives, one recovers the component action:

$$S_{SIM} = m^2 \int \left(Z \bar{Z} - i \psi_+ \frac{1}{\partial_{++}} \bar{\psi}_+ \right); \quad (3.7)$$

the Lorentz invariant terms are of course unchanged. A general SIM-superspace action uses the measure $\int d_+ \bar{d}_+$; the Lagrangian is constructed out of the SIM-superspace superfields $Z, \psi_-, \bar{Z}, \bar{\psi}_-$ (3.4,3.5) in such a way that the net weight of $+$ and $+$ indicies is minus one and the $-$, $-$ indices enter only in scalar combinations. Thus, for example, we find novel terms such as⁶

$$S_{new} = -im \int d_+ \bar{d}_+ \left(d_+ Z \frac{1}{\partial_{++}} \psi_- \right) + c.c., \quad (3.8)$$

which gives rise to

$$S_{new} = m \int \left(\psi_+ \frac{\partial_{-+}}{\partial_{++}} \psi_+ - Z F - \psi_+ \psi_- \right) + c.c.. \quad (3.9)$$

We have not studied the physical consequence of these novel mass terms, but note that they resemble Majorana masses and thus cannot arise for charged fields⁷.

⁶This is SIM(2) invariant if $d_+ Z$ and ψ_- transform as the components of a single Weyl spinor, as follows from (3.1,3.4).

⁷For completeness, we give their four-component form; the fermionic terms are: $\psi_+ \frac{\partial_{-+}}{\partial_{++}} \psi_+ - \psi_+ \psi_- = \psi_+ \frac{1}{\partial_{++}} (\partial_{-+} \psi_+ - \partial_{++} \psi_-) \propto \bar{\psi} \frac{\not{\partial}}{n \cdot \partial} \psi_L$.

4 Local forms of the SIM(2) action

One may introduce unconstrained complex auxiliary SIM-superfields X^{++} to remove the nonlocality as follows:

$$S_{local} = im^2 \int d_+ \bar{d}_+ \left(X^{++} \partial_{++} \bar{X}^{++} + X^{++} Z + \bar{X}^{++} \bar{Z} \right); \quad (4.10)$$

Upon integrating out X^{++} , this gives the nonlocal action (3.6). In four-component notation, this becomes

$$S_{local} = -m^2 \int d^2 \theta_{SIM} (X n \cdot \partial \bar{X} + X Z + \bar{X} \bar{Z}), \quad (4.11)$$

where $d^2 \theta_{SIM}$ is the SIM-superspace measure $\bar{D} \not{n} D \propto (\bar{\zeta} D_L)(\bar{\zeta} D_R)$, and X transforms so as to ensure the SIM(2) invariance of the action.

5 Coupling to gauge multiplets

The coupling to gauge multiplets is completely straightforward. The Lorentz invariant gauge multiplet action is unchanged; we couple to matter fields by making the replacement

$$Z \frac{1}{n \cdot \partial} \bar{Z} \rightarrow Z \frac{1}{n \cdot \nabla} \bar{Z} \quad (5.1)$$

in the SIM(2) part of the action. Then we have

$$\begin{aligned} S_{SIM} &= -im^2 \int d_+ \bar{d}_+ \left(Z \frac{1}{\nabla_{++}} \bar{Z} \right) \\ &\equiv -im^2 \int \nabla_+ \bar{\nabla}_+ \left(Z \frac{1}{\nabla_{++}} \bar{Z} \right) \\ &= m^2 \int \left(Z \bar{Z} - i \psi_+ \frac{1}{\nabla_{++}} \bar{\psi}_+ \right). \end{aligned} \quad (5.2)$$

An interesting question arises whether there are SIM(2) Lorentz-noninvariant terms possible for the gauge fields themselves. The answer, surprisingly, appears to be yes. We define SIM(2) superfields $W_\alpha, f_{+-} - iD' \equiv \nabla_- W_+, f_{--} \equiv \nabla_- W_-$ and their complex conjugates; here $f_{\alpha\beta} \propto F_{\mu\nu}^+ \gamma^{\mu\nu}$ is the self-dual part of the gauge field strength in two-component notation. Then we may write down terms such as

$$S_{nonlocal} = -m^2 \int d_+ \bar{d}_+ \left(W_+ \frac{1}{\nabla_{++}^2} \bar{W}_+ \right). \quad (5.3)$$

Defining components $\lambda_{\pm} \equiv W_{\pm}$, $f_{++} \equiv \nabla_+ W_+$ and the θ -independent components of D' , $f_{+-} = f_{-+}$, f_{--} , we find

$$S_{nonlocal} = m^2 \int \left(-i\lambda_+ \frac{1}{\nabla_{++}} \bar{\lambda}_+ + f_{++} \frac{1}{\nabla_{++}^2} \bar{f}_{++} \right) . \quad (5.4)$$

In four component notation, these become

$$S_{nonlocal} = m^2 \int (\bar{d} \not{n} d_L) \left(\bar{W} \not{n} \frac{1}{(n \cdot \nabla)^2} W \right) , \quad (5.5)$$

and

$$S_{nonlocal} = m^2 \int \left(\bar{\lambda} \not{n} \frac{1}{n \cdot \nabla} \lambda_L - n^\mu n^\nu F_{\mu\rho} \frac{1}{(n \cdot \nabla)^2} F_\nu{}^\rho \right) , \quad (5.6)$$

respectively. A remarkable simplification occurs when we make the lightcone gauge choice $n \cdot A = 0$: the entire nonlocality drops out of the gauge fields, and the second term in nonlocal action (5.4,5.6) reduces to the usual mass term A^2 , supplemented with the gauge condition⁸ $n \cdot A = 0$. This makes it seem unlikely that theory can be renormalizable; nevertheless, it is surprising that one can write down a gauge invariant mass term in four dimensions, albeit one that is only SIM(2) invariant.

One may wonder if the gauge system truly violates Lorentz symmetry; for the fermionic term, the stress-tensor has nonsymmetric terms, and thus does not generate Lorentz transformations [3]. One can do a similar calculation here, but there is a simpler argument: Lorentz invariance implies that massive vectors have three degrees of freedom; the massive vectors constructed here have only two degrees of freedom⁹. Unfortunately, the same argument almost certainly implies that this mass term is not a phenomenologically viable alternative to the Higgs mechanism. In particular, the equivalence theorem¹⁰ [5] implies that in the lightcone gauge, the longitudinal modes of the gauge bosons can be described by the couplings of scalar goldstone fields, and such fields are not present when one introduces a mass for the vector bosons by a Lorentz noninvariant but SIM-invariant term such as (5.6).

All the work in [1, 2, 3], as well as ours, implicitly assumes that $n \cdot \partial \neq 0$; as emphasized to us by Erik Verlinde, this implies a number of subtleties—in particular, modes annihilated by $n \cdot \partial$ do not become massive, and in the gauge $n \cdot A = 0$, the mass term is not quite A^2 , but rather has an implicit projection to leave a residual gauge-invariance under gauge transformation with parameters ω obeying $n \cdot \partial \omega = 0$.

⁸Of course, the usual massive vector field is not gauge invariant, and is not constrained by this condition, which drastically changes the physics

⁹As for massless fields, the gauge condition $n \cdot A = 0$ eliminates one degree of freedom, and the field $\tilde{n} \cdot A$ becomes a nondynamical Lagrange multiplier; since all representations of SIM(2) are one-dimensional, there is no reason to expect the usual (Lorentz-invariant) counting of states for massive fields.

¹⁰We thank George Sterman for pointing out the relevance of this theorem.

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References

- [1] A. G. Cohen and S. L. Glashow, *A Lorentz-violating origin of neutrino mass?* arXiv:hep-ph/0605036.
- [2] A. G. Cohen and S. L. Glashow, *Very special relativity*, arXiv:hep-ph/0601236.
- [3] A. G. Cohen and D. Z. Freedman, *SIM(2) and SUSY*, arXiv:hep-th/0605172.
- [4] D. Z. Freedman, private communication.
- [5] J. M. Cornwall, D. N. Levin and G. Tiktopoulos, *Derivation Of Gauge Invariance From High-Energy Unitarity Bounds On The S-Matrix*, Phys. Rev. D **10**, 1145 (1974) [Erratum-ibid. D **11**, 972 (1975)].
C. E. Vayonakis, *Born Helicity Amplitudes And Cross-Sections In Nonabelian Gauge Theories*, Lett. Nuovo Cim. **17**, 383 (1976).
B. W. Lee, C. Quigg and H. B. Thacker, *Weak Interactions At Very High-Energies: The Role Of The Higgs Boson Mass*, Phys. Rev. D **16**, 1519 (1977).